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Policy-Relevant Nonconvexities in the Production of Multiple Forest Benefits[1]

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This paper challenges common assumptions about convexity in forest rotation models which optimize timber *plus* nontimber benefits. If a local optimum occurs earlier than the globally optimal age, policy based on marginal incentives may achieve suboptimal results. Policy-relevant nonconvexities are more likely if (i) nontimber benefits dominate for young stands while the optimal age depends primarily on timber benefits, or (ii) nontimber benefits dominate for mature stands and also determine the optimal age. Nonconvexities may create either temporary or persistent difficulties. Policymakers may improve efficiency by exploiting the relationship between the timber-only optimum and the global optimum.

INTRODUCTION

Resource economists and forest managers focus much attention on the optimal timing of timber harvests (see Johansson and Löfgren [12], Newman [16]). However, federal legislation[2] calls for public land managers to produce a variety of

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[2]For example, the Multiple Use-Sustained Yield Act of 1960 and the Forest and Rangeland Renewable Resources Planning Act of 1974 as amended by the National Forest Management Act of 1976.

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outputs. In response, public forest managers increasingly focus on the economics of multiple use [3, 10]. In addition, some owners of private forestland consider noncommercial benefits related to outdoor recreation (Max and Lehman [15]).

A "forest benefits function" conceptually measures the benefits which society or firms receive from a variety of forest outputs. Previous analyses of multiple-use forestry (e.g. [3, 10, 13]) assume that the forest benefits function is convex, a hypothesis which enables simple, cost-effective policy tools to succeed. This paper examines the hypothesis of convexity. The objectives are (i) to establish that the forest benefits function may include nonconvex regions for a wide range of circumstances, (ii) to characterize nonconvexities which cause simple policy tools to fail, and (iii) to recommend appropriate policy options when policy-relevant nonconvexities exist.

Private landowners will produce an efficient mix of timber and nontimber outputs if paid (optimal) linear subsidies per acre held in various age classes [13], while public managers can produce an efficient mix if their analysis sets equivalent linear shadow prices for each age class [3]. However, linear shadow prices reliably achieve the efficient mix of forest outputs only if individual forest stands interact to produce nontimber goods through a concave joint production function [13]. Considering the complexity of biophysical production functions, an assumption of concavity could prove unreasonable and a nonconcave production function could induce nonconvexity in the forest benefits function. Heuristically, the addition of nontimber benefits to the manager's objective function might create a relevant nonconvexity since the biophysical production process may not translate into a convex function for human benefits.

Following convention [2, 4, 6], the term nonconvexity refers to characteristics of the forest production and benefit functions which might cause a violation of the second-order conditions for optimization. While Lewis and Schmalensee [14] examine nonconvexities arising due to fixed harvest costs, the current paper focuses on nonconvexities intrinsic to biophysical production. Applying Davidson and Harris's [6] classification, the Lewis–Schmalensee analysis treats a "transitional nonconvexity" while this paper studies a "stock nonconvexity" because the timber stock, which corresponds to a stand's age, generates the nonconvex objective function. Here, the cyclical nature of forest rotation models is central to whether nonconvexities are policy relevant.

As in environmental economics [2, 4], a counterfactual convexity assumption may have severe consequences. Since these consequences include a potential reduction of the benefits which society receives from forest resources, economists should examine the convexity assumption. But in practice, this examination may substantially increase the cost of analyzing trade-offs among forest outputs. For example, the alternative to linear programming might be a case-by-case comparison of a large number of discrete management regimes. This paper identifies some conditions which favor nonconvexities in forest benefits.

The next section develops a tractable foundation through the single stand problem for which the planning horizon extends for only one harvest cycle. The paper then extends the analysis to the standard case involving an infinite planning horizon, where delays in the first harvest cycle incur the opportunity costs of delays in future harvest cycles. Finally, the paper draws conclusions. Through each successive level of complexity, the paper identifies the relevance of and practical solutions to nonconvexities.

THE SINGLE STAND, SINGLE ROTATION BASELINE

This section uses Hartman's [10] single stand, single rotation model to develop the basic analysis of nonconvexities in the forest benefits function. The manager's objective is to maximize the present value of total, multiple-use benefits received from a single stand planted at time zero and harvested at age T, including both timber benefits, $B(T)$, and nontimber benefits, $A(T)$,

$$\max_T e^{-rT}[B(T) + A(T)] = \max_T e^{-rT}\left[pf(T) + \int_0^T e^{r(T-t)}a(t)\,dt\right], \quad (1)$$

where $B(T)$ is the timber volume available at age T, $f(T)$, times unit price, p; $A(T)$ is the sum of the annual nontimber benefits received at rate $a(\cdot)$, compounded as timber grows to age T; and r is the discount rate. Timber price remains exogenous and $a(\cdot) \geq 0$.[3] This model omits forestry costs, a simplification which leaves the main results unaffected. Using a prime (′) to denote differentiation, a locally optimal harvest age, T^*, satisfies

$$pf'(T^*) + a(T^*) = rpf(T^*), \quad (2)$$

with the second-order condition that

$$pf''(T) + a'(T) < rpf'(T) \qquad \text{at } T = T^*. \quad (3)$$

Intuitively, the age at which the marginal total benefit from a small delay in harvest equals the marginal opportunity cost of delay defines a local optimum.

Problem (1) may include sources of nonconvexity which are not unique to multiple-use forestry. First, nonconvexities may arise if price varies (perhaps discretely) with the age, the diameter, or the quality of trees. Second, if timber grows according to a logistic or an S-shaped growth curve, then $f(T)$ may induce a concave region in timber benefits, $B(T)$, below the inflection point at $T_{f'}$; mathematically, $T_{f'}$ maximizes the marginal growth rate, $f'(T)$ (i.e., $f''(T_{f'}) = 0$, $f'''(T_{f'}) < 0$). Typically, analysts assume that rotation ages greater than $T_{f'}$ compose the relevant range. This paper assumes a constant price and discusses nonconvexity due to $f(T)$ only in relation to multiple use.

If the second-order condition (3) holds for all forest ages, $T > 0$, then (2) is sufficient as well as necessary for a global optimum. However, if the timber growth rate or the flow of nontimber benefits rises (i.e., $f''(T) > 0$ or $a'(T) > 0$) for some $T > 0$, then (2) may not identify a unique optimum. Multiple optima are possible whenever timber and/or nontimber benefits are concave over some forest ages, such as if $T_{f'} > 0$ (as usual) or if the growth rate of nontimber benefits exceeds the

[3] In most cases, one may define nontimber outputs as positive benefits, rather than as costs avoided, relative to some background forest good. For example, if young stands are highly erodible and erosion diminishes trout populations in local streams, $a(\cdot) > 0$ could represent a trout benefit that increases with stand age, rather than $a(\cdot) < 0$ representing lost soil protection.

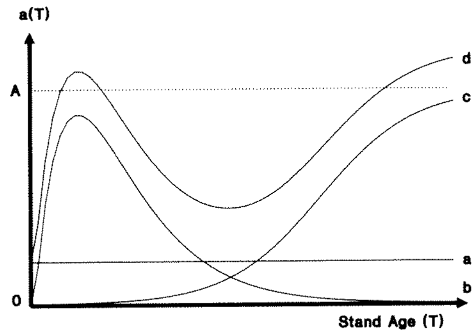

FIG. 1. Generic nontimber benefit curves, $a(T)$, of types a through d. Curve c may approach an asymptote such as $a(\cdot) = A$.

discount rate $(a' / a > r)$ for some T.[4] Heuristically, if nontimber benefits rise rapidly enough over some range of ages, then nontimber benefits can create a nonconvex objective function. This paper critically examines nonconvexities which may arise from a variety of forms for nontimber benefits (e.g. [5, 7, 9, 10, 17]).

Existence of Nonconvexities

This subsection establishes that the a priori rationale for assuming convexity in the forest benefits function is weak. Generic examples of nontimber benefits, $a(\cdot)$ (see Fig. 1, Table I), identify plausible cases when nontimber benefits in (1) are likely to include a nonconvex region and, therefore, a potential violation of (3). Empirical examples of these generic nontimber benefit curves, where letters refer to Fig. 1, are (a) an existence value derived from the maintenance of open, but not necessarily wild, spaces; (b) wildlife species best adapted to early successional/plantation stages of forests or forage production for cattle grazing; (c) wildlife species (including trout) adapted to mature forests or greater scenic beauty of old growth forests; and (d) bird species diversity or small game hunting when the primary species changes with forest age (see [5; 9, pp. 79–84, 317–320; 17; 18]). Modifications of these generic curves (Fig. 1) can generate $a(\cdot)$ for virtually any nontimber benefit. For example, by shifting the peak in curve b to age zero, the curve would represent water yield benefits (Table I; see [7]). Or, a parabolic form for curve b (Table I), shifted to the right, represents benefits from a wildlife species best adapted to middle-aged forests. Curve d (Fig. 1; Table I) may fit a particular nontimber good (like turkey habitat, see [9, p. 319]) or may represent a composite of multiple nontimber benefits; as illustrated in Fig. 1, curve d is the vertical sum of curves a–c so that here curve d represents a composite of several nontimber benefits. For selected functional forms, Table I identifies when nontimber benefits contribute a nonconvex component in (1) (i.e., when $a'/a > r$).

[4] $A(T)$ is concave for all T when $a'(T)/A'(T) > -r$, which holds for $a(T) > 0$ and $a'(T) > 0$; these conditions existed under Hartman's [10] assumptions, although his analysis assumed that (3) held at a unique solution for (2). If $a'(T)/a(T) > r$, then $e^{-rT}A(T)$ is nonconvex.

TABLE I
Selected Functional Forms for Nontimber Benefits

Type[a]	Annual benefits ($a(T)$)	Parameter restrictions and comments	Nonconvex[b] range of $e^{-rT}A(T)$	Examples[c]
a	a_0	No restrictions. No unique peak.	Convex for all T.	Existence of open space.
b	$\beta Te^{-b_1T} + k$	$\beta > 0$, $b_1 \geq 0$. For $k = 0$, peaks at $T_a = 1/b_1$.	Concave for $T < \beta/(r + \beta b_1)$.	Grazing or rabbits for peak in 5–10 years. Water yield for peak in 0–1 years.
	$b_2 - b_0(T - b_1)^2$	For $0 < b_1 - \sqrt{(b_2/b_0)} < T < b_1 + \sqrt{(b_2/b_0)}$; $b_0, b_1, b_2 > 0$. Otherwise, $a(T) = 0$. Peak at $T_a = b_1$.	Concave for $1 + rb_1 - \sqrt{(1 + r^2b_1/b_0)} < rT < 1 + rb_1 + \sqrt{(1 + r^2b_1/b_0)}$.	Ruffed grouse for peak ≈ 20 years.
c	$K/(1 + e^{c_0 - c_1T})$	$K > 0$, $c_1 > 0$. No peak, asymptote at K.	For $c_1 > r$, concave for $T < [c_0 + \ln(c_1/r - 1)]/c_1$. Otherwise convex.	Spotted owl, red-cockaded woodpecker, squirrels, scenic view.
	$K(T - t_0)^\alpha$	$T > t_0$, $0 < \alpha \leq 1$. For $T < t_0$, $a(T) = 0$. No peak or asymptote.	Concave for $T < 1/r$.	Wilderness (for t_0 large).
d	Combination of above.	Multiple peaks and troughs possible.	Concave whenever $a'(T)/a(T) > r$.	Bird species diversity, turkey.

[a]Types correspond to illustrations in Fig. 1. Curves b and c in Fig. 1 illustrate the first examples in the table.
[b]The present value of the nontimber benefits received over one rotation, $e^{-rT}A(T)$, is concave for T satisfying $a'(T)/a(T) > r$.
[c]Heuristic examples based on published sources [5, 7, 8, 17, 18].

Given the marginal conditions in (2), Fig. 2 portrays representative examples of nonconvexities which may arise in the single stand, single rotation problem. As previously noted, condition (2) involves a marginal benefit to delaying the harvest (MBD) and a marginal opportunity cost of delaying the harvest (MOC), defined as

$$\text{MBD} = pf'(T) + a(T), \qquad \text{MBD}|_{a=0} = pf'(T);$$
$$\text{MOC} = rpf(T). \tag{4}$$

Then, by (2) and (3), local optima, T^*, occur wherever MBD equals MOC and MOC intersects MBD from below. In contrast, the timber-only optimum age, T^{B*}, occurs where $\text{MBD}|_{a=0}$ equals MOC. Denote the global optimum age as T^{**}. While $\text{MBD}|_{a=0}$ exhibits a unique peak at the inflection point in a logistic timber-growth curve, MBD may exhibit multiple peaks and troughs (Fig. 2), each

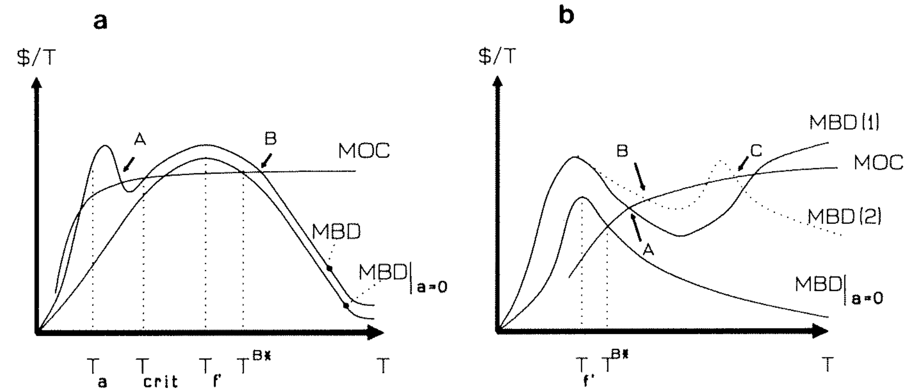

FIG. 2. Graphical solution of the single stand, single rotation problem.

of which corresponds to an inflection point in the present value of the forest benefits curve, Eq. (1).

The existence of multiple local maxima depends on the location and the height of the peak in nontimber benefits (Fig. 1) relative to the location and the height of the peak in $\mathrm{MBD}|_{a=0}$ and relative to the height of the MOC curve (Fig. 2). The examples illustrated in Fig. 2 assume that the timber volume curve is nondecreasing, so that MOC does not decline before leveling off. If, for example, mortality reduces merchantable timber volume in older stands, any additional nonconvexities would be similar to those described here.

Figure 2a involves nontimber benefits exemplified by grazing or water yield benefits (b in Fig. 1, Table I). Let T_a denote the age at which the largest annual flow of nontimber benefits occurs (so $a'(T_a) = 0$ and $a''(T_a) < 0$ for some $T_a \leq \infty$). Assuming that nontimber benefits follow the first type b function (Table I), the curves drawn in Fig. 2a assume that T_a occurs well before the inflection point in the timber volume curve ($T_a = 1/b_1 < T_{f'}$).[5] Clearly, as the age of maximal nontimber benefits (T_a) moves farther from this inflection point ($T_{f'}$), MBD is more likely to include multiple peaks. Considering timber benefits only, the global optimum presumably occurs at T^{B*}, although an earlier local optimum is possible if MOC and $\mathrm{MBD}|_{a=0}$ satisfy conditions (2) and (3) at a younger age (not shown). Local optima in total benefits occur at points A and B (Fig. 2a), with ages respectively above and below the timber-only optimum at T^{B*}. Since finite T^{B*} exists, Fig. 2a applies to inframarginal timberlands (land with a positive net present value for timber only) and one local optimum for (1) must occur after T^{B*}; but if multiple optima occur, one local optimum must occur at an age less than T^{B*}.

This case (Fig. 2a) clarifies two points. First, when management objectives include nontimber benefits, especially as represented by curve b (Fig. 1), the rationale for assuming $T > T_{f'}$ as the relevant range becomes tenuous. Forest managers' traditional emphasis on volume maximization [11, pp. 169–173], possibly combined with or derived from experience in identifying global economic maxima,

[5]In Fig. 2a, T_a occurs just before the first peak in MBD because MBD includes $pf'(T)$.

has created an a priori consensus on the relevant range.[6] This consensus is equivalent to assuming that the rate of timber growth exceeds the discount rate ($f'/f > r$) for early ages ($T < T_{f'}$) so $\mathrm{MBD}|_{a=0}$ exceeds MOC. However, the addition of nontimber benefits suggests that managers at least reevaluate this consensus. Second, extramarginal timberlands, where timber-only management is not economic, are more likely to exhibit a local optimum at an early stand age. This local optimum might occur uniquely. However, if, for early ages, a delay in harvest incurs marginal opportunity costs greater than the marginal timber benefits obtained (so $\mathrm{MOC} > \mathrm{MBD}|_{a=0}$ for $T < T_{f'}$), nontimber benefits may raise the total marginal benefits high enough to create multiple local optima. Figure 2a illustrates this nonconvexity for a type b nontimber benefit.[7] Finally, a local optimum at an early age will most likely occur on extramarginal timberlands because, for most inframarginal lands of age $T < T_{f'}$, $\mathrm{MBD}|_{a=0}$ already exceeds MOC so that consideration of nontimber benefits only reinforces the incentives to delay the harvest.[8]

An intermediate case, exemplified by ruffed grouse (Table I; omitted from Fig. 2), involves a parabolic curve for nontimber benefits and T_a greater than $T_{f'}$ by just enough years to create a single peak in MBD just past $T_{f'}$. In this case, two local maxima may occur and one of these must occur after $T_{f'}$. If MOC exceeds MBD for some age T such that $T_{f'} < T < T^*$, where T^* is one local optimum, then both maxima may occur after $T_{f'}$. In this case a consensus that all relevant ages exceed $T_{f'}$ would not eliminate a nonconvexity problem, even when this assumption is valid. Whether the timberland is infra- or extramarginal does not affect this result.

Nontimber benefits of type c, such as spotted owls or scenic views (Fig. 1; Table I), may generate nonconvexities similar to those discussed previously; however, the present discussion focuses on nonconvexities which are unique to type c amenity benefits. Figure 2b portrays two qualitatively different situations involving type c amenity benefits. In both situations, annual type c amenity benefits rise rapidly at some age which greatly exceeds $T_{f'}$. This rapidly rising segment of $a(\cdot)$ determines the results because the type c amenities create a MBD curve with two peaks, one near $T_{f'}$ and the second at a much older forest age (Fig. 2b). As before, the first situation (solid-lined, MBD(1) curve, Fig. 2b) does not necessarily involve a nonconvexity at finite ages because MOC may only rise above MBD at one age (Fig. 2b, point A). However, if MBD exceeds MOC as T goes to infinity (e.g., see MBD(1), Fig. 2a), then preservation, with $T^* = \infty$, becomes a candidate for the global optimum policy even though $e^{-rT}A(T)$ remains finite in the limit. However, for the second situation (dash-lined, MBD(2) curve, Fig. 2b), the second peak in MBD occurs just above MOC so that two local maxima may occur after $T_{f'}$ (Fig. 2b, points B and C). The second optimum occurs because the second peak in MBD(2) (Fig. 2a) is low enough and early enough that MBD(2) falls below MOC as the marginal timber benefits from delaying the harvest ($\mathrm{MBD}|_{a=0}$) decline.

[6]If the manager acts as if $a(\cdot) = 0$, the relevant range involves ages above the minimum age for marketable trees ($T : p > 0$).

[7]This situation may arise for type a benefits, but only as an artifact of the finite horizon assumption. Intuitively, $a(\cdot) = a_0$ should result in $T^{**} = T^{B*}$, as is easily verified for the indefinite horizon model below, because nontimber benefits simplify to a constant with present value a_0/r.

[8]More complicated models (see below), where nontimber benefits enter both sides of the first-order condition, weaken this conclusion.

As previously noted, type d nontimber benefits (Fig. 1) can represent a composite of multiple nontimber outputs (e.g., bird species diversity, Table I). Consistent with intuition, type d amenities can generate all the nonconvexities discussed above. Which of these local maxima actually occurs depends on the stand ages at which the peaks in $a(T)$ are high enough and the troughs in $a(T)$ are low enough (curve d, Fig. 1) that the MBD curve falls below MOC, producing multiple solutions to (2)–(3).

Relevance and Solutions

In the single stand, single rotation problem, Eq. (2) provides the basis for myopic or adaptive policies, including taxes or subsidies and linear programming procedures (e.g., [13]). Following Arrow [1], "myopia" describes policies which attempt to improve or optimize social welfare by looking only at short-run incentives quantified in a first-order, necessary condition. Myopic policies assume that front-line managers possess information only on current marginal conditions for each stand. For example, a myopic subsidy policy would pay $a(T)$ ($/acre) to owners of forest acres which are not harvested at age T.[9] A "relevant nonconvexity" prevents a myopic policy from achieving the optimum rotation age because managers harvest at the first age which satisfies (2)–(3) (the first local maximum) while these myopic managers overlook any global maximum at a later solution to (2). If the first maximum is global, any nonconvexity problems remain irrelevant.

If a global maximum occurs at T^{**}, an earlier local maximum (a), say $T^* < T^{**}$, might block a myopic policy/procedure from achieving the global maximum. In the application of a myopic subsidy ($a(\cdot)$) to owners of growing timber, the incentive to cut at the first local maximum must be offset by, for example, offering a "lump subsidy" to landowners who agree to hold their timber in a "forest conservation reserve" beginning at age T^* until age T_{crit} ($T^* < T_{\text{crit}} \leq T^{**}$), where T_{crit} is the minimum age at which (1) is again convex, so the myopic marginal benefits and costs lead the landowner to the next optimum age (which, in this example, is global). A conservation reserve program could contract landowners to hold timber to age T^{**}, but choosing the contract length involves a trade-off between budgeting a larger lump subsidy and the advantages of resuming a myopic subsidy schedule for stands older than T_{crit}.[10] If T^{**} occurs at B then Fig. 2a illustrates an example where T_{crit} is set equal to the stand age at which a local minimum occurs between the two local maxima at A and B.

Finally, the global maximum may involve preservation—i.e., an infinite harvest age—with nonconvexities creating a local maximum at some finite age. Then resource policymakers must adjust a myopic policy/procedure, as discussed above, and then be prepared to provide $a(\cdot)$ from T_{crit} onward. This situation is more likely if, contrary to the assumptions above, the forest manager begins with a stand already at some positive age.

[9] If the owner does harvest at age T, then the owner has received subsides with a compounded value of $A(T)$.

[10] The reserve program may be simpler, but the myopic policy provides flexibility and probably is more robust under imperfect information. This issue remains for future analysis.

THE SINGLE STAND, INDEFINITE HORIZON PROBLEM

With an indefinite planning horizon, the forest manager weighs the benefits of the current rotation against the opportunity costs of delaying benefits from each future rotation [8, 10, 12, 16]. In the usual formulation, the manager begins with bare land, plants an even-aged stand,[11] and maximizes the present value of forest benefits:

$$\max_T \left[e^{-rT}(B(T) + A(T)) - C \right] / \left[1 - e^{-rT} \right]. \tag{5}$$

This formulation explicitly incorporates regeneration costs, C, which occur at the start of each rotation. These costs were fixed in, and therefore omitted from, the single rotation model. Furthermore, this problem reduces to the familiar Faustmann [8] problem when $A(T) = 0$ for all T.

Objective (5) produces Hartman's [10] necessary condition for an optimum rotation age, T^*,

$$pf'(T^*) + a(T^*) = \{r/(1 - e^{-rT})\}\left[B(T^*) + e^{-rT}A(T^*) - C \right], \tag{6a}$$

or equivalently,

$$\left[pf'(T) + a(T) \right]\{(1 - e^{-rT})/r\} = \left[B(T) + e^{-rT}A(T) - C \right] \text{ at } T = T^*, \tag{6b}$$

which are represented, respectively, as

$$\text{MBD} = \text{MOC}_\infty \tag{6'a}$$

$$\text{PVMBD} = \text{RTH}. \tag{6'b}$$

The left-hand side of (6a) is the marginal benefit of delay (MBD) of the first harvest. On the right-hand side of (6a), the term in braces is an adjusted discount rate, where the adjustment accounts for the opportunity cost of delaying future rotations (see [10, 16]). Using this adjusted discount rate leads to interpretation of the left-hand side of (6b) as a present-value MBD (PVMBD) curve. Returning to (6a), the term in brackets represents the return to harvest and regeneration (RTH), including the immediate timber benefits minus regeneration costs plus the amenity benefits received over the next growing period. Then the right-hand side of (6a) is the marginal opportunity cost to delay (MOC_∞) curve, given an indefinite (∞) planning horizon.

For model (5)–(6), a myopic tax policy, or its linear programming analog, would pay $a(T)$ to holders of age T timber and pay $\{r/(1 - e^{-rT})\} \cdot e^{-rT}A(T)$ to forest

[11] The assumption of a bare-land starting point eliminates a complication relevant to the indefinite horizon model. If the resource manager is initially endowed with a stand of age S, then the objective function becomes

$$\max_{W,T} : pf(W + S)e^{-rW} + \int_S^{W+S} e^{-r(t-s)}a(t)\,dt + e^{-rW}\frac{\left[e^{-rT}(B(T) + A(T)) - C \right]}{(1 - e^{-rT})},$$

where W is the time of the first harvest and T is the Hartman–Faustmann rotation. If S is old enough, and depending on $a(\cdot)$, the optimal W could prove infinite even if T is finite.

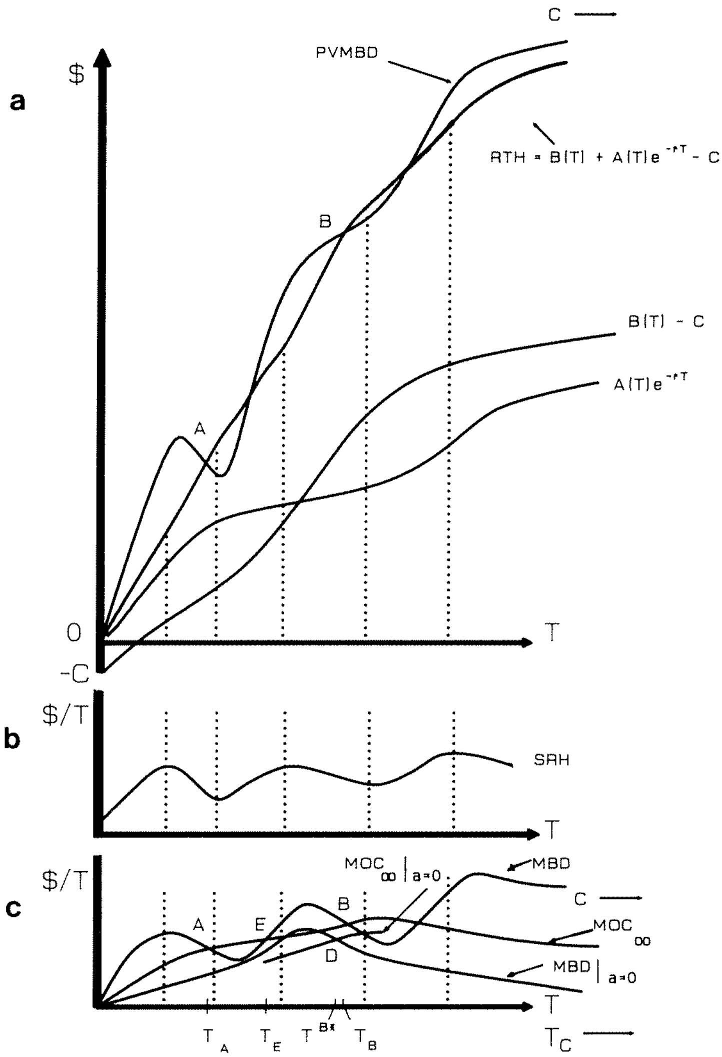

FIG. 3. Graphical solution of the single stand, infinite horizon problem. Acronyms are defined by Eqs. (6) and (6′), except for $MOC_{\infty}|_{a=0}$, which represents MOC_{∞} when managers ignore nontimber benefits. T^{B*} corresponds to point D in panel *c*.

owners who harvest at age T. A relevant nonconvexity prevents such a policy or program from achieving the optimum mix of outputs.

Existence of Nonconvexities

Given (6b), a graphical approach again facilitates discussion of nonconvexities. Solutions to (6a) satisfy the second-order condition if the MBD curve intersects the MOC_{∞} curve from above [10]; a similar relationship must hold for PVMBD and RTH in (6b).

These relationships are illustrated in Fig. 3. The appendix fully develops Fig. 3, where the role of panel b is discussed. This section provides an overview of Fig. 3, with particular attention to panel a. Discussion of panel c follows the case study.

Figure 3 assumes a type d nontimber benefits function (Fig. 1) which captures results for other types in Fig. 1. Figure 3a applies to (6b) and shows the PVMBD curve with the RTH curve, $B(T) + e^{-rT}A(T) - C$, and its components. Figure 3c summarizes necessary condition (6a). The dotted lines in Fig. 3 identify inflection points in the return to harvest curve (see RTH). Local maxima for objective (5) occur at A and B (Fig. 3a or 3c). Label C suggests that an infinite timber age may represent a "local" maxima (Fig. 3a or 3c).

Here nonconvexities (i.e., multiple local maxima) arise in much the same manner as in the single stand, single rotation problem. However, amenity values now appear on the right-hand side of the necessary condition (cf. (2) and (6)). Heuristically, these amenity values create a "waviness" in the return to harvest function and the MOC_{∞} function (Figs. 3a and 3c). This waviness increases the likelihood of multiple intersections of, for example, the PVMBD curve and the RTH curve (see Fig. 3a). Therefore, if nonconvexity problems are likely in the single stand, single rotation model, these problems are even more likely in management of multiple rotations.

An Illustrative Case Study

The foregoing analysis provides theoretical grounds for the existence of nonconvexities. This subsection examines a plausible case study: timber production and cattle grazing on forestland in western Montana. Timber yield data were published by the USDA Forest Service [19, Appendix B-7D, p. 40] while grazing yields are documented in the Lolo National Forest Planning Library (input files for the FORPLAN model). Lolo National Forest managers use these data for analyses required under the National Forest Management Act of 1976. The data pertain to moderately dry sites populated by mixed ponderosa pine (*Pinus ponderosa*) and Douglas fir (*Pseudotsuga menziesii* var. *glauca*) on soils which are not highly erodible. Data included the timber volume (thousand board-feet, mbf, per acre) and grazing potential (animal unit months, aum, per acre-year) for stands at each age.

The case study employed simple functional forms for $f(T)$ and $a(T)$. The timber yield model assumes a carrying capacity, K, of 15.055 mbf/acre[12] and timber grows according to a logistic curve,

$$f(T) = K/(1 + e^{[\alpha - \Theta T]}). \tag{7a}$$

Let $z = f(T)/K$ define the proportion of carrying capacity attained by age T, ordinary least squares (OLS) regression estimates the parameters using

$$\begin{aligned} \ln([1 - z]/z) &= 6.1824 - 0.0801T, \qquad R^2 = 0.88, \\ &\ (1.603) \quad (0.0073), \qquad N = 18, \end{aligned} \tag{7b}$$

where parentheses provide standard errors. The model for grazing benefits assumes the first type b curve (Table I with $k = 0$), which OLS estimates as

$$\begin{aligned} \ln(a(T)/T) = \ln\beta - b_1 T &= 0.4323 - 0.0850T, \quad R^2 = 0.99, \\ &\ (0.1060) \quad (0.00014) \quad N = 9. \end{aligned} \tag{8}$$

[12]The carrying capacity is taken directly from the Lolo Forest's draft EIS [19, Appendix B-7D].

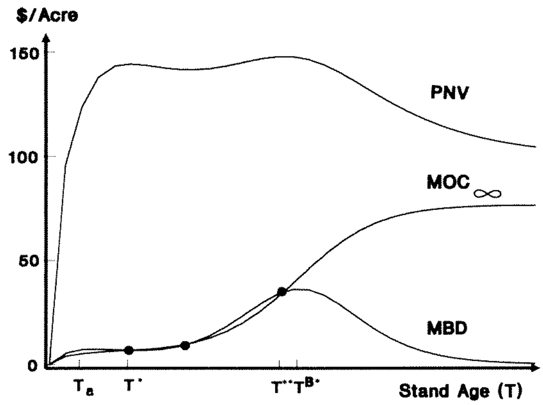

FIG. 4. Case study for western Montana timber and grazing. Ages T_a, T^*, T^{**}, and T^{B*} respectively correspond to 12.5, 26, 73, and 77.2 years.

This case study uses the United States Forest Service's 4% discount rate and values forest outputs at \$120/mbf for timber and at \$20/aum for grazing. Regeneration costs (C in (5)) are assumed zero.

Analysis of marginal conditions (MOC_∞, MBD) and the present net value (PNV, given by (5) with (7)–(8); Fig. 4) for this case identifies a local maximum of \$144/acre at age 26 (T^*) with a global maximum of \$148/acre at age 73 (T^{**}). These values contrast with a Faustmann rotation age of 76 years (T^{B*}), which would produce a PNV of \$147/acre, including grazing. In this case, the first local maxima and the Faustmann age both would produce $> 97\%$ of the PNV at T^{**}. However, in general these suboptimal rotation ages could produce a significantly smaller share of potential net benefits.

These results (Fig. 4) illustrate a policy-relevant nonconvexity because the marginal conditions (MOC_∞ and MBD) would lead myopic managers to the local, suboptimal maximum at age 26 = T^*. This nonconvexity occurs because a significant and sharp peak in annual nontimber benefits (\$6.66/acre at age 12.5 = T_a; from Table I with (8)) occurs relatively far from the peak in annual timber growth (\$36.2/acre at age 77.2 = $T_{f'}$ for (7)), so the convex regions of nontimber and timber benefits do not coincide. The policy relevance arises because the peak benefits from grazing do not make the first optimum age a global optimum.

Implications for Myopic Policies

The implications discussed for the single rotation model apply to nonconvexities in the indefinite horizon model as well, with minor modifications. In addition, the indefinite horizon model involves interactions between successive harvest cycles. The cyclical nature of the forestry problem provides a novel angle for analysis of nonconvexities.

As in pollution control problems (cf. [4]), nonconvexities may or may not be policy relevant, depending on the relationship between the no-policy equilibrium and the globally efficient equilibrium. Nonconvexities might cause a newly imposed, myopic policy to lead competitive equilibrium either to a local maximum or further below the global maximum. In forest management, these results may be either temporary or persistent. Nonconvexities may create a temporary set of problems until the first harvest after imposition of a new policy, but once the "bare land" condition occurs, nonconvexities either may misguide policy instruments no longer or may undermine policy initiatives even further. Figures 3c and 4 illustrate three possible scenarios.

Scenario 1. In Figure 3c, point D corresponds to the Faustmann rotation age, T^{B*}, which is optimal if forest managers ignore nontimber benefits. Now suppose that point A represents the global maximum, so $T^{**} = T_A$ when managers do consider nontimber benefits. If resource policymakers suddenly impose a new policy, some landowners/managers will be holding stands older than T_A, with plans to cut at T^{B*}. The optimal policy might be to allow these resource owners to cut at T^{B*}, after which time a myopic policy would lead them to a harvest cycle of period T_A.[13] This scenario involves a temporary problem with nonconvexities, but nonconvexities present no long-term problem. That is, after the first harvest, a myopic policy leads to optimal harvesting at T_A.

Scenario 2. Now suppose that point B (Fig. 3c) defines the global optimum, so now $T^{**} = T_B$. A new policy might convince forest owners to optimally delay their initial plan (T^{B*}) and to harvest instead at T_B. But after this initial harvest, the local maximum at T_A blocks successful implementation of a myopic policy. This scenario involves a temporarily successful myopic policy, but ultimately nonconvexity problems persist in the long term. One available solution is to modify the pure myopic policy by eliminating amenity-related subsidies before some forest age T_{crit} (not shown on Fig. 3c),[14] where $T_E < T_{crit} \leq T^{B*}$ and T_E marks the local minimum between T_A and the global optimum at T_B. Under this modification, the myopic marginal benefits and the opportunity costs of timber values alone would lead resource owners up to T_{crit}, at which time amenity-related subsidies begin and the myopic incentives lead resource managers to optimal harvesting at T_B. Of course if $T^{**} < T^{B*}$, as in the case example (Fig. 4), then $T_{crit} \leqslant T^{**}$ is required but the *modified* myopic policy still leads to harvesting at $T_B = T^{**}$ (73 years, in this case).

Scenario 3. Now suppose that the global optimum, T^{**}, occurs at some age T_C. If T_C is infinite, then the nonconvexity problems are temporary; the policy challenge is to cause resource owners to allow stands to reach the ages in region C (Fig. 3c), past all local maxima at finite forest ages. One response might be for the government to purchase these lands. However, if T_C is finite, then the nonconvexity problems persist for each harvest cycle, and policymakers must face the challenge of developing an effective modification or a replacement for a purely myopic policy or a linear programming procedure.

These three scenarios assume that the individual stands are profitable based on timber benefits alone. However, in some cases, inclusion of amenity benefits might bring previously uneconomic stands, where $T^{B*} = \infty$, into harvest management

[13]This discussion ignores the possibility that a new policy might alter the incentives for forest management, causing these forest owners to revise their harvest plan from T^{B*}. In practice, the policymaker should investigate the post-policy objective function of forest managers and how it might modify the time of first harvest. This caution also applies to extramarginal timberlands.

[14]Another available solution is a conservation reserve program of the type described for the single rotation case. In practice, eliminating subsides prior to T_{crit} reduces the required budget for subsides.

because T^{**} is finite [3, 10]. For these extramarginal stands, unfortunately, modifications to myopic policy cannot exploit the myopic incentives of a timber-only benefit function.[15]

In the case where T_C produces the global optimum (Fig. 3c), policymakers may exploit myopic incentives of the timber-only benefit function to bring forestlands up to T^{B*}. Then myopic payments, of $a(T)$ to delay harvesting on age T lands and of $\{re^{-rT}A(T)/(1 - e^{-rT})\}$ to induce harvesting on age T lands, would lead resource owners to T_B. Unfortunately, no myopic policy, based on actual amenity benefits, would lead resource owners from T_B to T_C.

However, a modified myopic policy, based on an artificial "amenity value function," $a_p(T)$, could prove feasible. Policymakers might successfully design and publicize this amenity value function for the purposes of public policy. Some criteria for $a_p(T)$ include

$$\int_0^T e^{-rt} a_p(t)\, dt = e^{-rT} A_p(T) = e^{-rT} A(T);$$

$$a_p(T^*) = a(T^{**}), \quad \text{where } T^{**} \text{ is the globally optimal age;}$$

$$\delta a_p(T^{**})/\delta T^{**} = \delta a(T^{**})/\delta T^{**}.$$

These criteria ensure that $a_p(T)$ will satisfy the necessary conditions (6) for optimization of the single stand, infinite horizon model. The first criterion[16] ensures that any subsidy based on $a_p(T)$ requires the same present-value budget that $a(T)$ justifies. If the convex region of $e^{-rT}A_p(T)$ encompasses the convex region of $e^{-rT}B(T)$, then this modified amenity function at least introduces no nonconvexity problems which were not present in $e^{-rT}B(T)$.

To this point, the paper focuses on the existence of nonconvexities and suggests policy responses. But if managers implement a purely myopic policy, are there any costs of ignoring nonconvexities other than failure to achieve the global maximum? The case study provides an example. A myopic policy leads to rotations at age 26, a local maximum, while ignoring nontimber benefits leads to rotations at age 76. In this instance, the myopic policy would actually *decrease* social net benefits by 2.0% relative to the Faustmann rotation policy at age 76 (Fig. 4), with the magnitude of total losses rising as the acreage of the relevant stand increases. Under the conditions of the case study, a policy choice to ignore nontimber benefits entirely, which guarantees suboptimal forestry benefits, is preferred to a myopic policy. Clearly, failure to recognize nonconvexities can, in some cases, reduce social benefits from levels produced when management ignores nontimber benefits.

CONCLUSIONS

Nonconvexities may arise in forest management simply because nontimber outputs, which humans value, depend on a biophysical production function; that is,

[15]A simple (but nonmyopic) policy could subsidize harvests at T^{**} by paying $\$X$ per acre, where X exceeds the manager's harvest costs and $X \leq e^{-rT}A(T)$.

[16]In the single rotation model, the second equals sign could be replaced by an inequality ($\leq$) because nontimber benefits only affect one side of the necessary condition (2) (cf. (6)).

the timber stock, as the primary input to biophysical production of nontimber goods, may induce nonconvexities in a forest manager's objective function (cf. [6]). More complex models only raise the chance that these nonconvexities will arise. Nonconvexities complicate policy analyses since analyses based on simple linear shadow prices become unreliable. These potential difficulties should not, however, rule out efforts to optimize forest management plans (cf. [4]).

Nonconvexities are more likely to occur if the age at which annual nontimber benefits peak differs substantially from the age at which the annual increment to timber benefits peaks, especially if nontimber goods produce benefits comparable to or greater than timber benefits. When increments to timber benefits dominate those to nontimber benefits from early stand ages, a nonconvexity will be policy relevant if the globally optimal rotation age significantly depends on timber benefits. The case study exemplifies these circumstances, where the global optimum occurs near the standard Faustmann age. On the contrary, when nontimber benefits dominate increments to timber benefits from late stand ages, a nonconvexity will be policy relevant if the globally optimal rotation age significantly depends on nontimber benefits. Any relevant nonconvexity challenges managers to bypass extrema which precede the globally optimal age. This challenge may recur with successive rotations.

Economists could design modified myopic policies. Modifications might exploit the relationship between the harvest age which maximizes timber benefits and the various harvest ages which produce local maxima in multiple-use benefits. Successful policies will encourage optimal forest management through a mix of actual marginal incentives for timber production and marginal subsidies for nontimber production which alter the incentives apparent to myopic landowners/managers. These modified policies apply primarily to inframarginal timberlands.

Future research might design policies for nonindustrial private land, where relevant nonconvexities (and externalities) might lead managers away from social optima. Such policies might exploit incentives from the nonindustrial landowner's utility function (see [15]) in a manner similar to the exploitation of timber incentives discussed above.

Policy analysts might expend significant resources in efforts to obtain the optimal solution despite nonconvexities. These costs should, of course, never exceed the potential gains. In some cases, an a priori decision to permit stand specialization may reduce analytical costs while raising the total of forest benefits realized over the status quo. In such cases, the Hartman model again provides a useful benchmark.

The economics literature for forest resources generally raises the nonconvexities issue only as a qualification to studies of multiple forest stands. Given current knowledge of stand interactions, the multi-stand model adds little to the foregoing results from single-stand models; a Hartman-type rotation age remains optimal, at least on average (see [3]). Stand interactions may raise or shift peaks in stand-level, nontimber benefit functions, thereby making nonconvexities in the multiple forest benefits function more likely. In the multi-stand problem, however, spatial separation of different-aged stands might mitigate nonconvexity problems (cf. [2]). These issues remain for future research, possibly within the context of "dominant-use management" as treated informally in recent National Forest planning.

This paper shows that crucial nonconvexities may arise given plausible parameter values, even at the simplest level of forest management. Policy analysts should always treat an assumption of a convex benefit function for multiple forest outputs with caution.

APPENDIX

This appendix develops Fig. 3. Differentiating the return to harvest (RTH, (6b)) determines the slope of RTH (SRH) (Fig. 3b):

$$\mathrm{SRH} = \delta B/\delta T + \delta\left[e^{-rT}A(T)\right]/\delta T = pf'(T) + a(T)e^{-rT}.$$

The extrema in SRH occur at alternating inflection points in RTH (Figs. 3a and 3b). The marginal benefit of delay (MBD in (6)) curve is nearly identical to SRH since SRH includes the discounted value of $a(T)$ while MBD omits the discount factor. Relative to SRH, the MBD curve rises with the forest age and its extrema occur to the right of the extrema of SRH (cf. Figs. 3b and 3c).

To convert the MBD curve (Fig. 3c) to PVMBD, divide (6a) by the adjusted discount rate, $Z(T)^{-1}$, where

$$Z(T) = (1 - e^{-rT})/r, \qquad Z'(T) > 0, \qquad Z''(T) < 0,$$
$$\lim_{(T\to 0)} Z(T) = 0,$$
$$\lim_{(T\to\infty)} Z(T) = 1/r.$$

Thus, multiplying MBD by $Z(T)$ expands MBD vertically and the expansion factor rises asymptotically to $1/r$, generating PVMBD (Fig. 3a). In general, PVMBD remains positive and may or may not include increasing or decreasing segments.

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